WAVES GENERATED AT AN INTERFACE BETWEEN TWO FLUIDS OF DIFFERENT DENSITIES BY THE MOTION OF A CIRCULAR CYLINDER AND A SYMMETRIC WING

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In a two-layer stably density-stratified fluid we have experimentally investigated the parameters of waves generated at the interface by the uniform horizontal motion of a circular cylinder and a symmetric wing profile in the upper layer. The measurements were carried out in the range of first Froude numbers $1 < \mathrm{Fr} \leqslant 10$:

$$\mathrm{Fr} = U^2(2+\varepsilon)/\varepsilon g R, \tag{1}$$

where U is the velocity of motion of the body; 2R, its maximum transverse dimension; $\varepsilon = \rho_2/\rho_1 - 1$; ρ_1, ρ_2, densities of the fluid in the upper and lower layers; and g, acceleration of gravity.

The experimental data were compared with the results of calculations based on the linear theory of internal waves in an inviscid stratified fluid [1-3]. It was found that the wavelengths are fairly well predicted by this theory. The same result has been obtained in earlier papers [4-6]. The calculation of the wave amplitudes within the scope of the theory of ideal fluids, on the other hand, yields values that are too low, because the additional perturbations associated with friction and the separation of flow from the body are ignored.

We attach a coordinate system to the body as shown in Fig. 1 (in the case of a circular cylinder its origin is situated at the center of the circle). All linear dimensions are referred to the characteristic length R of the body; these include the coordinate x, the deviation η of the interface from the rest state, the wavelength L and wave amplitude η_m, the thicknesses H_1 and H_2 of the upper and lower fluid layers, the distance h from the x axis to the interface, and the wave number $K = 2\pi/L$.

The experiments were carried out in a closed channel having transparent walls, a length of 120 cm, a width of 20 cm, and a height of 30 cm. The upper layer was tap water, and the lower layer a solution of glycerin in water, with a density from 1.005 to 1.015 g/cm^3. The circular cylinder had a radius R = 1 cm. The Reynolds number $\mathrm{Re} = 2RU/\nu$ was varied from 400 to 1500 in the cylinder experiments. The wing had the cross section shown in Fig. 1. The perturbation induced by it in an ideal fluid was simulated by a combination of a point source of strength q placed at the origin and a sink of the same total strength q distributed uniformly between points a_1 and a_2 [7]. In the experiments we used a wing with parameters $q/2\pi RU = 0.5$, $a_1/R = 0.5$, $a_2/R = 10.8$, $l/2R = 6$, and R = 0.5 cm. The Reynolds number in these tests was varied from 250 to 800.

The body was driven into uniform motion and stopped at the end of the path practically instantaneously (in comparison with the internal wave period). The lower fluid layer was dyed in the experiments. The wave parameters were determined from photographs taken at times when the wave pattern was stationary (in the coordinate system attached to the body) and before wave reflection could take place at the end walls of the channel. The random errors occurring in the determination of the wave parameters in the given tests were estimated according to the values of the coefficient of variation and turned out to be ≈15%.

According to the above-indicated theory, the waves at the interface become single-harmonic modes for large values of x:

$$\eta = \eta_m \sin(kx + \varphi). \tag{2}$$

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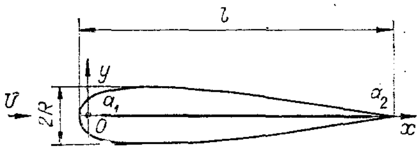

Fig. 1

The quantity k in (2) does not depend on h or on the shape of the body. It is determined by the dispersion relation [8]

$$\frac{U^2}{gR} = \frac{1}{k}\frac{\rho_2 - \rho_1}{\rho_1 \operatorname{cth} kH_1 + \rho_2 \operatorname{cth} kH_2}, \tag{3}$$

which for large values of H_1 and H_2 takes the following form on the basis of (1):

$$L = 2\pi/k = 2\pi \mathrm{Fr}. \tag{4}$$

The corresponding asymptotic expressions of this theory for the amplitudes η_m and the initial phase angles φ, valid for large x, h, H_1, and H_2, have the following form for the cylinder:

$$\eta_m = \frac{4\pi k}{2+\varepsilon} e^{-kh}, \quad \varphi = 0; \tag{5}$$

and for the wing:

$$\eta_m = \frac{2q}{UR(2+\varepsilon)} e^{-kh} \sqrt{A^2 + B^2}, \quad \varphi = \operatorname{arctg}(B/A), \tag{6}$$

where

$$A = 1 - \frac{\sin ka_2 - \sin ka_1}{k(a_2 - a_1)}; \quad B = \frac{\cos ka_2 - \cos ka_1}{k(a_2 - a_1)}.$$

The values of x, H_1, and H_2, beginning with which expressions (4)-(6) become valid, are estimated from the results of numerical calculations performed by V. A. Sukharev according to the method described in [3]. Thus, according to the calculations for a wing with Fr = 9, the wavelength determined from the distance between the first and second crests (counting from the body) differ only 0.9% from the asymptotic result, while the value of η_m determined at the first crest differs only 0.3% from the asymptotic value. For Fr = 3 equally small deviations occur in the wavelengths and amplitudes from their asymptotic values, beginning with the second crest. These computational results are consistent in order of magnitude with the experimental data.

The values of H_1 and H_2 were at least equal to 24 in the experiments. The wavelengths calculated on the basis of expressions (3) and (4) in this case differ less than 1% from one another for Fr = 10. For Fr < 10 the difference is even smaller. The calculations also show that the asymptotic expressions (5) and (6) for the wave amplitudes are also applicable with good accuracy in the experimental ranges of Fr, H_1, and H_2. The values of h at which expressions (5) and (6) begin to be valid can be estimated from the experimental data. This problem will be discussed more in detail below.

In tests with miscible fluids a certain smearing of the interface is inevitable. The influence of this factor was also investigated by numerical calculations performed on a computer by Sukharev according to the method of [3]. The function $\rho(y)$ in this case was estimated in terms of an integral error function with mean value corresponding to $y = -h$ and standard deviation σ. The influence of smearing of the interface is illustrated in Figs. 2 and 3, which gives, along with the solid curves corresponding to expressions (4) and (6), curves (dashed) obtained by numerical calculations for $\sigma/R = 0.167$, which is typical of the given experiments.

The most significant difference between the conditions of the experiments and the calculations according to expressions (3)-(6) is attributable to the viscosity of the real fluid. On the one hand, the viscosity causes wave attenuation. This influence of the viscosity is manifested only at large distances from the body, and in the present experiments fell within the experimental error limits. On the other hand, when a body is towed in a viscous fluid, a momentum equal to the viscous friction of the body is imparted to the flow, becoming particularly appreciable in the presence of flow separation from the body. This viscosity effect is the principal cause of the discrepancy between the calculated and experimental data for the wave amplitudes.

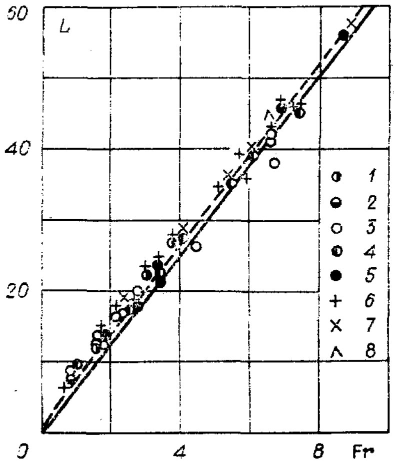

Fig. 2

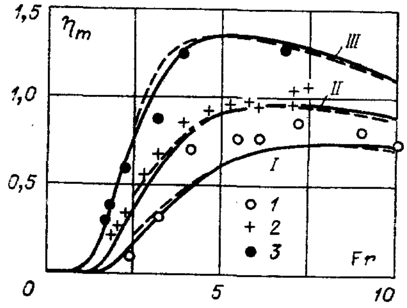

Fig. 3

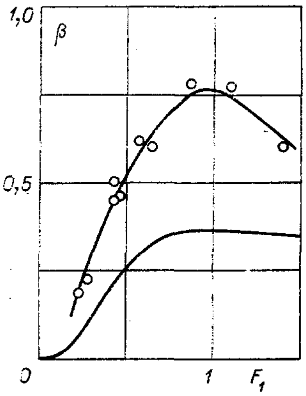

Fig. 4

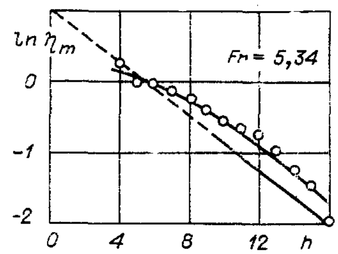

Fig. 5

The foregoing is illustrated in Figs. 2-5. Figure 2 gives the wavelength data. The numbers 1-3 identify experimental points obtained in tests with a cylinder towed over respective distances h = 2, 3, 4; the numbers 4-8 refer to a wing with respective values of h = 2, 4, 6, 8, 10.

The experimental and calculated wave amplitudes generated by the motion of a circular cylinder are compared in Fig. 4. In this case the substitution

$$\beta = \frac{2+\varepsilon}{4\pi}\eta_m h, \quad F_1 = \mathrm{Fr}/h$$

permits the experimental and calculated data for various values of Fr and h to be written in the form of functions of a single variable:

$$\beta_e = \beta_e(F_1),\ \beta_c = \beta_c(F_1).$$

However, due to the above-mentioned viscosity effect, the functions β_e (upper curve in Fig. 4) and β_c (lower curve) differ considerably fron one another. The fact that the experimental points in Fig. 4 fit a single universal curve corroborates the exponential behavior of the variables η_m as a function of h for a cylinder down to h = 2.

In the case of a wing profile the exponential dependence of η_m on h is corroborated in the experiments only for h > 10, i.e., for h roughly greater than l. Consequently, the transformation of the function of two variables into a function of a single variable, as is also possible for expression (6), does not impart suitable universality to the experimental data over the entire investigated range of h. For a wing profile the experimental wave amplitudes are compared with the calculated values in Figs. 3 and 5. Figure 3 gives η_m as a function of Fr for h = 8, 6, 4 (points 1-3 and curves I-III, respectively), while Fig. 5 gives η_m as a function of h for Fr = 5.34. It is evident from Figs. 3-5 that in the case of the poorly streamlined circular cylinder the experimentally obtained wave amplitudes can exceed the calculated values by a factor of more than two. The discrepancy between the experimental and calculated data for the more streamlined wing profile does not exceed 25%.

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ONE FORM OF THE EQUATIONS OF HYDRODYNAMICS OF AN IDEAL INCOMPRESSIBLE FLUID AND THE VARIATIONAL PRINCIPLE FOR NONSTEADY FLOW WITH A FREE SURFACE

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In the investigation of nonsteady flows having a free surface there are well-known difficulties [1] connected with the formulation of the problems in the traditional statements of Euler or Lagrange.

Using the "Clebsch potentials" χ, μ, and λ one can write the equations for an ideal incompressible fluid in the form [2, 3]

$$\partial v_i/\partial x_i = 0; \tag{1}$$

$$\partial\mu/\partial t + v_i\partial\mu/\partial x_i = 0; \tag{2}$$

$$\partial\lambda/\partial t + v_i\partial\lambda/\partial x_i = 0, \tag{3}$$

where the velocity components v_i are expressed by the equations

$$v_i = \partial\chi/\partial x_i + \lambda\partial\mu/\partial x_i \quad (i = 1, 2, 3). \tag{4}$$

Here and later in writing the equations we use the rule of summation over double repeated ("dummy") indices.

For the pressure p there is the expression

$$p = -\rho\left(\frac{\partial\chi}{\partial t} + \lambda\frac{\partial\mu}{\partial t} + \frac{1}{2}v_i^2\right) \quad (i = 1, 2, 3), \tag{5}$$

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